

# May We Expect CP- and T-Violating Effects in Neutrino Oscillations?

TUD-IKTP/99-01

1 Feb 1999

K. R. Schubert  
Institut für Kern- und Teilchenphysik  
Technische Universität Dresden  
Schubert@physik.tu-dresden.de

**Abstract:** Neutrino oscillations with three families of leptons are described with the help of a unitary neutrino mixing matrix  $\mathbf{U}$  in analogy to the Standard Model quark mixing with the CKM matrix. If  $\mathbf{U}$  contains a nontrivial phase and if all three neutrino mass eigenstates have a different mass, there will be CP- and T-violating asymmetries at distances  $L$  which are large compared to  $E/D^2$ , where  $D^2$  is the largest difference of neutrino mass squares. We give explicit expressions for these  $L$ -dependent asymmetries in a frame which is able to describe the present solar and atmospheric oscillation observations.

The recent strong evidence for neutrino oscillations in atmospheric neutrino production [1] has amplified the interest in the basic properties of these oscillations. The observed strength of the neutral weak interaction of atmospheric neutrinos, in the  $\pi^0/e$  ratio [2], does not require the assumption of sterile neutrinos but allows a complete description in an extended Standard Model of elementary particles.

Without changing anything in the principles of the model, the extension can easily be made by adding three SU(2) singlets of right-handed neutrinos —  $\nu_{eR}$ ,  $\nu_{\mu R}$ , and  $\nu_{\tau R}$  — to the set of elementary fermions. This leads to a fourth mass matrix  $\mathbf{M}_\nu$  in addition to  $\mathbf{M}_\ell$ ,  $\mathbf{M}_Q$ , and  $\mathbf{M}_q$  for charged leptons  $\ell$ , up- and down-type quarks  $Q$  and  $q$ . The diagonalization of  $\mathbf{M}_\ell$  leads to the Yukawa term  $m_e \bar{e}e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau$  in the Lagrangian, and  $\mathbf{M}_\nu$  cannot be diagonalized at the same time. The SU(2) partners of  $e$ ,  $\mu$ , and  $\tau$  have to be linear superpositions of mass eigenstates  $\nu_1$ ,  $\nu_2$ , and  $\nu_3$ . In very close analogy to the CKM matrix of quark mixing [3, 4], the right-handed neutrino singlet fields require a unitary transformation  $\mathbf{U}$  between the two triplets in family space,

$$\nu' = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \text{ and } \nu = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$

For experimental convenience,  $\mathbf{U}$  should be defined for the neutrinos and not for the charged leptons, i. e.

$$\nu' = \mathbf{U} \nu, \quad \mathbf{U} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}, \quad \nu = \mathbf{U}^\dagger \nu', \quad \bar{\nu}' = \mathbf{U}^* \bar{\nu}, \quad \bar{\nu} = \mathbf{U}^T \bar{\nu}',$$

in contrast to the quarks where it is completely arbitrary if  $\mathbf{V}$  describes mixing of  $q$  and  $q'$  with mass-diagonal  $Q$  or mixing between  $Q$  and  $Q'$  with diagonal  $q$  fields. Since the

first publication of Kobayashi and Maskawa [4] we are used to the choice  $q' = \mathbf{V}q$ , but the opposite convention is preferred for leptons since only their charged components can be directly detected.

The extended Standard Model has 25 free parameters. In addition to the conventional 18, these are

$$m(\nu_1), m(\nu_2), m(\nu_3), \vartheta_{e2}, \vartheta_{\mu3}, \vartheta_{e3}, \text{ and } \Delta,$$

where we have parametrized  $\mathbf{U}$  as

$$\mathbf{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\mu3} & s_{\mu3} \\ 0 & -s_{\mu3} & c_{\mu3} \end{pmatrix} \begin{pmatrix} c_{e3} & 0 & s_{e3}e^{-i\Delta} \\ 0 & 1 & 0 \\ -s_{e3}e^{i\Delta} & 0 & c_{e3} \end{pmatrix} \begin{pmatrix} c_{e2} & s_{e2} & 0 \\ -s_{e2} & c_{e2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with  $0 \leq \vartheta_{ij} \leq \pi/2$ ,  $c_{ij} = \cos \vartheta_{ij}$ ,  $s_{ij} = \sin \vartheta_{ij}$ , and  $0 \leq \Delta < 2\pi$  in complete analogy to the parametrisation of  $\mathbf{V}$  by the Particle Data Group [5]. If all three masses  $m(\nu_i)$  are different from each other, if all three mixing angles are different from 0 and from  $\pi/2$ , and if  $\Delta$  is different from 0 and from  $\pi$ , it is impossible [6] to get a real matrix  $\mathbf{U}$  by rotating lepton field phases. The extended Standard Model then gets the possibility of showing T-violating and CP-violating effects in the lepton sector, the strengths of which are proportional to the phase-rotation invariant quantity [6]

$$J^{(\nu)} = s_{\mu3} s_{e3} s_{e2} c_{\mu3} c_{e3}^2 c_{e2} \sin \Delta .$$

For any  $i, f, k, l$  (without summing), this quantity is given by [6]

$$J^{(\nu)} = \text{Im}(U_{ik}U_{fl}U_{il}^*U_{fk}^*) \cdot \sum_{\alpha, \beta} \epsilon_{if\alpha} \epsilon_{kl\beta} .$$

T-and CP-violating effects require these phase-rotation invariant combinations of four mixing matrix elements in the pertinent amplitudes. We investigate their possible presence in the following.

At its time of production by a W-boson, a neutrino is given by

$$\nu'_i = \sum_{j=1}^3 U_{ij} \nu_j$$

with  $i = e, \mu$ , or  $\tau$ . After a distance  $L$ , this neutrino disappears by creation of a charged lepton  $f = e, \mu$ , or  $\tau$ . The probability for this process is

$$P(i \rightarrow f) = P_{fi} = \left| \sum_{j=1}^3 U_{ij}U_{fj}^* e^{-im_j^2 L/2E} \right|^2.$$

At small distances,  $P_{fi} = \delta_{fi}$  because of  $\mathbf{U}\mathbf{U}^\dagger = 1$ . We now order the mass eigenstates by increasing mass and abbreviate

$$m_2^2 - m_1^2 = d^2, \quad m_3^2 - m_1^2 = D^2.$$

Present observations by Super-Kamiokande [1] and by the various solar neutrino disappearance experiments [7] indicate

$$d^2 \approx 5 \cdot 10^{-6} \text{eV}^2 \ll D^2 \approx 3 \cdot 10^{-3} \text{eV}^2 ,$$

where the value for  $d^2$  follows from the best fit with the small angle solution including the MSW effect [8].

For a given neutrino energy  $E$ , the first distances with observable oscillations are of the order  $L = o(L_2)$  with

$$L_2 = E/D^2.$$

At these distances, we can approximate  $d^2 \approx 0$  and get

$$P_{fi} = |U_{i1}U_{f1}^* + U_{i2}U_{f2}^* + U_{i3}U_{f3}^* e^{-iD^2L/2E}|^2.$$

Because of the unitarity of  $\mathbf{U}$ , this is

$$\begin{aligned} P_{ii} &= |(1 - |U_{i3}|^2) + |U_{i3}|^2 e^{-iD^2L/2E}|^2 \\ &= 1 - 4|U_{i3}|^2(1 - |U_{i3}|^2) \sin^2(D^2L/4E), \end{aligned}$$

and for  $f \neq i$

$$\begin{aligned} P_{fi} &= |U_{i3}|^2|U_{f3}|^2|1 - e^{-iD^2L/2E}|^2 \\ &= 4|U_{i3}|^2|U_{f3}|^2 \sin^2(D^2L/4E). \end{aligned}$$

Both expressions, for  $f = i$  and  $f \neq i$ , depend only on two mixing matrix elements and not on terms of the form  $U_{ik}U_{fl}U_{il}^*U_{fk}^*$ . Hence, there are no CP- or T-violating observations at distances  $L = o(L_2)$ . Because of  $|U_{ij}|^2 = |U_{ij}^*|^2$  we have

$$P_{\overline{fi}} = P_{fi} \quad \text{and} \quad P_{if} = P_{fi} ,$$

where  $P_{\overline{fi}}$  denotes the probability for the transition  $\overline{\nu}_i \rightarrow \overline{\nu}_f$ . The situation changes if  $L$  increases and approaches

$$L_3 = E/d^2 .$$

Here we have

$$\begin{aligned} P_{fi} &= |U_{i1}U_{f1}^* + U_{i2}U_{f2}^* e^{-id^2L/2E} + U_{i3}U_{f3}^* e^{-iD^2L/2E}|^2 \\ &= \sum_{j=1}^3 |U_{ij}|^2|U_{fj}|^2 + 2\text{Re}[U_{i1}U_{f2}U_{i2}^*U_{f1}^* e^{id^2L/2E} \\ &\quad + U_{i1}U_{f3}U_{i3}^*U_{f1}^* e^{iD^2L/2E} + U_{i2}U_{f3}U_{i3}^*U_{f2}^* e^{i(D^2-d^2)L/2E}] \end{aligned}$$

which contains three obviously CP-violating contributions if  $f \neq i$ . The disappearance experiments give CP-symmetric results; for  $f = i$  we have

$$P_{ii} = \sum_{j=1}^3 |U_{ij}|^4 + 2|U_{i1}|^2|U_{i2}|^2 \cos \frac{d^2L}{2E} + 2|U_{i1}|^2|U_{i3}|^2 \cos \frac{D^2L}{2E} + 2|U_{i2}|^2|U_{i3}|^2 \cos \frac{(D^2 - d^2)L}{2E}$$

$$= 1 - 4[|U_{i1}|^2|U_{i2}|^2 \sin^2 \frac{d^2 L}{4E} + |U_{i1}|^2|U_{i3}|^2 \sin^2 \frac{D^2 L}{4E} + |U_{i2}|^2|U_{i3}|^2 \sin^2 \frac{(D^2 - d^2)L}{4E}]$$

with the property

$$P_{\bar{i}i} = P_{ii} .$$

For further discussion of the CP-asymmetries in the appearance experiments, it is convenient to introduce factors  $p_{ifm}$  and phases  $\alpha_{ifm}$  with  $m = 1, 2, 3$ , defined as

$$U_{ik}U_{fl}U_{il}^*U_{fk}^* = \sum_{m=1}^3 \epsilon_{klm} p_{ifm} e^{i\alpha_{ifm}} .$$

The real parts are  $p_{ifm} \cos \alpha_{ifm}$ , depending on  $i$  and  $f$ , but independent of the sequence of  $ijkl$ . The imaginary parts are  $+J^{(\nu)}$  for  $ijkl = e\mu 12$  and all cyclic permutations, but  $-J^{(\nu)}$  for all other permutations. In terms of these factors and phases, the appearance rates are

$$\begin{aligned} P(i \rightarrow f) &= P_{fi} = \sum_{j=1}^3 |U_{ij}|^2 |U_{fj}|^2 + 2 \sum_{m=1}^3 p_{ifm} \cos\left(\frac{d_m^2 L}{2E} + \alpha_{ifm}\right) \\ &= -4 \sum_{m=1}^3 p_{ifm} \sin \frac{d_m^2 L}{4E} \sin\left(\frac{d_m^2 L}{4E} + \alpha_{ifm}\right) , \end{aligned}$$

with  $d_1^2 = D^2 - d^2$ ,  $d_2^2 = D^2$ , and  $d_3^2 = d^2$ . The CP, T, and CPT symmetry properties of these rates are easily seen; we obtain

$$P_{\bar{f}i} = P_{if} = -4 \sum_{m=1}^3 p_{ifm} \sin \frac{d_m^2 L}{4E} \sin\left(\frac{d_m^2 L}{4E} - \alpha_{ifm}\right) ,$$

which is different from  $P_{fi}$  if at least one of the three phases  $\alpha_{ifm}$  is different from 0 and  $\pi$ . In that case, CP and T are violated, but CPT is conserved.

When we approach  $L = o(L_3)$ , we have to average over two cosine terms if  $D^2 \gg d^2$  and if the energy resolution is limited. We obtain in this case

$$P_{fi} = \sum_{j=1}^3 |U_{ij}|^2 |U_{fj}|^2 + 2 p_{if3} \cos\left(\frac{d^2 L}{2E} + \alpha_{if3}\right) .$$

In contrast to  $L = o(L_2)$ , as shown above, there remain CP and T violating effects at  $L = o(L_3)$ .

It is straightforward to construct T- and CP-violating asymmetries. For the general case with  $L$  between  $L_2$  and  $L_3$ , they are

$$A_{fi} = \frac{P_{fi} - P_{if}}{P_{fi} + P_{if}} = \frac{P_{fi} - P_{\bar{f}i}}{P_{fi} + P_{\bar{f}i}} = \frac{\sum_{m=1}^3 p_{ifm} \sin \alpha_{ifm} \sin(d_m^2 L/4E) \cos(d_m^2 L/4E)}{\sum_{m=1}^3 p_{ifm} \cos \alpha_{ifm} \sin^2(d_m^2 L/4E)} .$$

For  $if = e\mu, \mu\tau$ , and  $\tau e$ , the three contributions  $p_{ifm} \sin \alpha_{ifm}$  in the numerator are equal to the invariant quantity  $J^{(\nu)}$  of the neutrino CKM matrix, leading to

$$A_{fi} = J^{(\nu)} \frac{\sum_m \sin(d_m^2 L/4E) \cos(d_m^2 L/4E)}{\sum_m p_{ifm} \cos \alpha_{ifm} \sin^2(d_m^2 L/4E)}.$$

For  $if = \mu e, \tau\mu, e\tau$ , the asymmetries have, obviously, the opposite sign. All asymmetries  $A_{fi}$  depend on the same property  $J^{(\nu)}$  of the neutrino CKM matrix, a result which can be found with the same simplicity in the CP violating effects of the meson sector [6].

Present experiments [1, 7] have still very large errors on  $\vartheta_{e2}$ ,  $\vartheta_{\mu3}$ , and  $\vartheta_{e3}$ ; but they permit to set an upper limit on  $J^{(\nu)}$ . With  $s_{\mu3} \approx c_{\mu3} \approx 1/\sqrt{2}$ ,  $s_{e2} \approx 0.04$ , and  $s_{e3} < s_{e2}$ , we obtain

$$|J^{(\nu)}| < 8 \cdot 10^{-4},$$

whereas the quark sector has the property  $J \approx +3 \cdot 10^{-5}$  [9] if CP violation in  $K^0\bar{K}^0$  oscillation has its origin in a complex quark CKM matrix. The small upper limit on  $J^{(\nu)}$  requires a large number of events in future neutrino oscillation experiments. This paper does not aim to estimate numbers of events. Such an estimate would require, even without assuming anything on the phase  $\Delta$ , better knowledge on the angles  $\vartheta_{e2}$  and  $\vartheta_{e3}$ .

## Acknowledgement

I would like to thank W. Hampel (MPI Heidelberg) und J. Urban (TU Dresden) for several helpful discussions.

## References

- [1] Y. Fukuda et al. (Super-Kamiokande), Phys. Rev. Lett. 81 (1998) 1562
- [2] T. Kajita (Super-Kamiokande), presented at NEUTRINO 98, Takayama (1998), hep-ex/9810001
- [3] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531
- [4] M. Kobayashi and T. Maskawa, Progr. Theor. Phys. 49 (1973) 652
- [5] Review of Particle Properties, Particle Data Group, Eur. Phys. J. C 3 (1998) 1
- [6] for a review, see e. g. C. Jarlskog, "CP Violation", Advanced Series on Directions in High Energy Physics, World Scientific Singapore (1989)
- [7] J. N. Bahcall, P. I. Krastev, and A. Yu. Smirnov, Phys. Rev. D 58 (1998) 096016
- [8] L. Wolfenstein, Phys. Rev. D 17 (1978) 2369  
S. P. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. 42 (1986) 913
- [9] using  $\eta$  from a fit of S. Mele, CERN-EP/98-133 (1998)